The effects of turbulence characteristics on sphere drag

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Further results are reported in determining the drag coefficient of spheres in terms of the turbulence characteristics of the flow in which they are immersed. C_D contours are given on plots of turbulence scale versus intensity and show distinct regions where artificially low and high drag can be experienced. The results are shown to be in accordance with theory based on published results for flow conditions upstream and downstream of the spheres.

A Strouhal number (St) based on turbulence macroscale and the r.m.s. value of the fluctuating component of axial velocity is shown to be very appropriate in this application.

Keywords: sphere drag; turbulence intensity; macroscale; Strouhal number

Introduction

For many years there has been a need for more accurate data on the drag coefficient of spheres immersed in highly turbulent flows. The two extremes of the Reynolds number (Re) range have provided few problems in this respect, since at very high values the sphere boundary layers are definitely turbulent and supercritical values of C_D may be taken from data sheets, whereas at the lower end of the scale "Stokes flow" applies and viscous effects predominate. At the intermediate Reynolds numbers, however, from roughly 2×10^3 to 2×10^5, the value of C_D is known from many published sources to be closely dependent on the turbulence characteristics of the approaching flow. The problems encountered by designers in such fields as solids conveying, particle dispersion and hydrocyclones are therefore obvious; straightforward C_D versus Re data sheets are rendered useless.

The pioneering work of Torobin and Gauvin[1] and Clamen and Gauvin[2] showed how very sensitive was the critical Reynolds number to turbulence intensity, the well known C_D-dip occurring at astonishingly low Re values for relative intensities of 30 or 40%, the sort of levels commonly encountered in pipe flows. Neve and Jaafar[3] extended this work to higher Reynolds numbers and found that their C_D values lined up very closely with those of the previous authors. These combined results showed that, for given turbulence intensities, $C_D = 0.3$ (the usual criterion for specifying Re_{crit}) could occur at up to three separate Reynolds numbers, leading the present authors to conclude that turbulence macroscale was also involved as a controlling parameter. Results have subsequently been published giving evidence for that suggestion.[4]

Remaining inconsistencies were assumed to be the result of, inter alia, the different tubulence spectra of the upstream flows, especially as early on Van der Hegge Zijnen[5] had reported sudden changes in heat transfer rates from cylinders when the characteristic frequency of turbulence of the oncoming flow was comparable with the von Karman shedding frequency in the downstream wake. More recently, confirmation has been published[6,7] that spheres also have a characteristic shedding frequency; so, Van der Hegge Zijnen's suggestion that a form of resonance can be experienced when the two frequencies are comparable is thought to be applicable to spheres also.

The current program[8] has therefore been aimed at investigating sphere drag by monitoring C_D changes in flows of known (measured) turbulence characteristics.

Turbulent flow approaching a sphere

Flows of high turbulence levels for the drag testing of spheres have usually been achieved using severe velocity gradients, including those produced by grids and jet edges. Davies, Fisher, and Barratt[9] and Fisher and Davies[10] were amongst the first to show systematically that underlying, coherent, nearly-regular structures existed in such shear layers and that these had the characteristics of axial arrays of vortices. They showed that close to the axis of a jet the convection velocity of these vortices was about 70% of the local fluid speed whereas in the outer jet region it could be considerably greater than the local speed.

In a subsequent paper, Lau, Fisher, and Fuchs[11] showed that the vortices were separated (streamwise) by about 1.26 times the nozzle diameter (or slot width) that produced the jet and that they were convected downstream at about 60% of the jet speed at nozzle exit. Further published results have tended to confirm these figures, although Lau[12] reported finding two distinct vortex streets; the major one (the subject of previous papers) converged on the jet centerline, whereas a street of minor vortices diverged from it. The spacing between major vortices was about $\lambda/D = 1$ at $x/D = 1.5$, increasing to $\lambda/D = 3$ at $x/D = 5$. The Strouhal number ($= fD/u_j$) decreased from about 0.8 at $x/D = 1$, $r/D = 0.5$ to about 0.45 at downstream stations.

Acton[13] investigated a mathematical model of vortices in the outer regions of a circular jet and found a convection velocity of $u_c/u_j = 0.56$ with a local fluid velocity $U/u_j = 0.65$. The Strouhal number (0.47) was very close to the value of 0.45 which constitutes a representative value from a number of published papers.

The general conclusion here then must be that spheres immersed in a turbulent shear layer are subject to a fairly regular and coherent vortex street of Strouhal number 0.45, being conveyed downstream at about 70% of the local fluid speed.

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Notation

C_D	Drag coefficient $(=\text{drag}/[\rho U^2\pi d^2/8])$
C_1	Constant (Equation 1)
d	Sphere diameter
D	Nozzle diameter (or width)
f	Frequency
I	Relative turbulence intensity $(=u'/U)$
L_x	x-direction macroscale of turbulence
r	Radius from jet axis
St	Strouhal number
Re	Reynolds number $(=Ud/\nu)$
$\mathrm{Re}_{\mathrm{crit}}$	Critical Reynolds number $(C_D=0.3)$
U	Local fluid speed (time mean value)
u'	r.m.s. fluctuating component of streamwise velocity
u_c	Convection speed of vortices
u_j	Fluid speed in nozzle
x	Downstream distance from nozzle exit or grid position
ν	Kinematic viscosity
λ	Spacing between approaching vortices

Flow downstream of the sphere

The phenomenon of shed vorticity behind a sphere has been a rather more contentious issue. The Strouhal number of 0.21 for two-dimensional cylinders has been well established, but the mechanism of vortex shedding from a sphere will obviously be different because of axisymmetry. The localized boundary layer separation will clearly not be stable and the earliest separation point is likely to rotate about the undisturbed flow axis, giving vorticity of a helical nature. In fact, on purely theoretical grounds, the wake is more likely to consist of two contrarotating helices since the flow upstream of the sphere will generally have zero net vorticity about the streamwise axis.

As early as 1938, Möller[14] investigated the flow behind a sphere and quoted a Strouhal number at $\mathrm{Re}=10^4$ almost ten times greater than for a cylinder. Much later, Mujumdar and Douglas[15] placed a hot wire in the wake and registered a value of $\mathrm{St}=0.2$ for $5600\leqslant\mathrm{Re}\leqslant 11{,}600$; that is, about the same as for a cylinder. Further systematic testing was reported by Achenbach[6] who used a water channel for lower Reynolds numbers and wind tunnels for higher ones. Over certain Reynolds number ranges, he was able to repeat some of Möller's results but his most important finding in the present context was to show that a characteristic Strouhal number did exist, over a wide Re range

$$\mathrm{St}=0.19\text{–}400/\mathrm{Re} \qquad [6000\leqslant\mathrm{Re}\leqslant 2\times10^5]$$

It should be noted that Achenbach's sensors were embedded in the sphere's surfaces at 75 degrees from the stagnation point, not in the wake. He reported an anticlockwise rotation (looking downstream) of the earliest separation point, which he ascribed to the driving fan direction. The shed vorticity was helical in nature and definitely not in the form of toroidal vortex rings.

In a very recent paper, Monkewitz[7] has investigated the instability boundaries for helical vortex production behind bluff bodies. Comparison is made with published experimental results and agreement is quite good when allowance is made for different normalizing techniques. His most surprising result is that for $\mathrm{Re}\geqslant 3300$ (based on wake diameter), large scale helical vortex shedding may be driven by a self-excited oscillation in the wake; that is, a "wavemaker" is active just downstream of the sphere and the shed vorticity is not a response to continuous feeding from an upstream disturbance such as a separating boundary layer.

At supercritical Reynolds numbers, Achenbach was not able to detect vortices using the given equipment, although a shedding process did reappear above $\mathrm{Re}=5\times10^6$.

The general conclusion for flow downstream of a sphere must therefore be that spheres do indeed have a characteristic shedding frequency giving a Strouhal number slightly less than 0.2 in the Reynolds number range of interest here.

Dynamic conditions at the sphere

So far, conditions upstream of a sphere have been defined in terms of the physical model of vortices approaching at a frequency dependent upon fluid speed and nozzle size but an equally valid Strouhal number could just as easily be defined, based on the eddy conditions themselves; that is, in terms of u' and L_x. The characteristic frequency would then be given by

$$f_1=\mathrm{St}_1(u'/L_x)$$

The wake conditions, on the other hand, give a frequency

$$f_2=\mathrm{St}_2(U/d)$$

If Van der Hegge Zijnen's suggestion that a type of resonance occurs when these two frequencies are comparable is correct, giving marked changes in heat transfer and (by implication) drag coefficient, then

$$\mathrm{St}_1(u'/L_x)\simeq\mathrm{St}_2(U/d)$$

or

$$\frac{L_x}{d}\simeq\frac{\mathrm{St}_1}{\mathrm{St}_2}\cdot\frac{u'}{U}\simeq\frac{\mathrm{St}_1}{\mathrm{St}_2}\cdot I \tag{1}$$

Thus, on a graph of L_x/d versus I, a low drag region might be expected along the line $(L_x/d)=C_1 I$ if the two Strouhal numbers (or their ratio) remain effectively constant.

Experimental arrangements

The experimental rig used for this work was the same as that fully described in Refs. 4 and 8, so only a brief indication will be given here.

An open-section wind tunnel of working section 405 × 240 mm produced an airflow of turbulence intensity less than about 0.4% at up to 47 m/s. Downstream turbulence for sphere drag testing was produced by three different grids and its characteristics were shown to conform to other published data on grid-generated turbulence. Testing was not attempted in the region close to the grids where "grid shadow" might still have been evident.

Spheres were of diameter 37.7 mm and mounted on a strain-gauged cantilever for drag force measurement, calibrated against static loads. The traversing gear on which this cantilever arrangement was mounted enabled the spheres to be positioned on the wind tunnel jet centerline at distances up to 2 m from the exit plane.

Air speeds were measured using a constant temperature hot-wire anemometer (CTA) and pitot-static tube, the former being calibrated against the latter. Turbulence intensity was measured using the r.m.s. and digital voltmeters on the CTA and macroscale was determined using a tunable bandpass filter to produce data for a power spectral density technique. This method is described in detail in Engineering Sciences Data Unit Item 74031 (*Aerodynamics*, Vol. 6).

For this project, a data logger was added to the previous rig to monitor five channels and compute instantaneous values of Re, I, and C_D. An on-site terminal then sent these data to the University's Gould supermini computer at 1.2 kilobaud for storage. The vast quantity of information available from over 2000 data points required the use of a Lotus 1–2–3 spreadsheet technique for analyzing results using prespecified bandwidths for Re, I, and L_x/d. "Macros" (stored keystrokes) were embedded within the spreadsheets to minimize the repetitive workload involved in analyzing data.

The traversing gear was stable and massive but a check was made to ensure that the natural frequency of the cantilever mounting did not correspond to any vortex shedding frequencies. This was found to be about 35 Hz.

Accuracy of measurement was considered to be as in Ref. 4. The least accurate figures relate to L_x because of the indirect way in which it is always determined. However $\pm 15\%$ is not disastrous when general guidelines are being sought. The accuracy of C_D measurement was as poor as $\pm 8\%$ but only at the lowest Reynolds numbers; at higher values, the accuracy obviously improved in inverse proportion to the square of Reynolds number. Velocity measurements (and therefore Reynolds number) were accurate to a few percent at the bottom of the range but far less than 1% at the top.

Discussion of results

The drag coefficient results are here plotted as contours on axes of scale versus turbulence intensity, a representation found useful in a previous paper.[4] In each case, the contours have been drawn by eye over an underlay of C_D spot values derived from the results spreadsheet, with selection being made using embedded macros against specified bandwidths of I, L_x/d and Re. Also, the "standard curve" C_D value for each Reynolds number has been placed at the origin since this justifiably occurs on each figure as a zero intensity and zero scale asymptotic value. Computer codes such as GINOSURF were found to be unacceptable for contouring because, although sufficient data points were obtained in each of the Figures 1–5, their spread across the rectangle of each graph was insufficiently uniform.

At the highest Reynolds numbers achievable with the current apparatus, Figure 1 shows that the standard curve value of

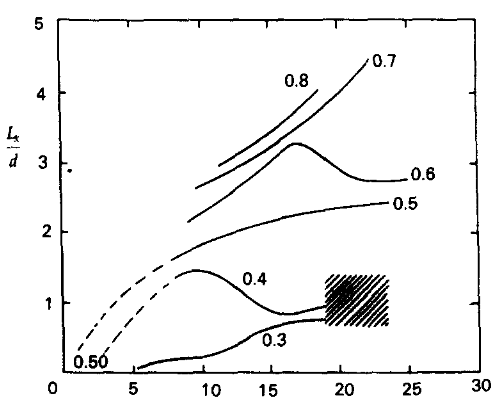

Figure 1 Drag coefficient contours: Re = 70,000 (approx.). Broken lines indicate lack of data; shaded zone indicate unstable results (see text)

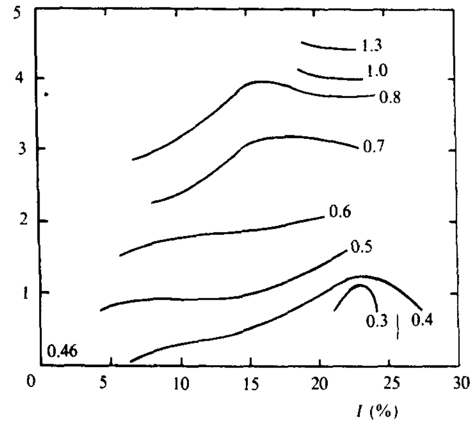

Figure 2 Drag coefficient contours: Re = 40,000 (approx.)

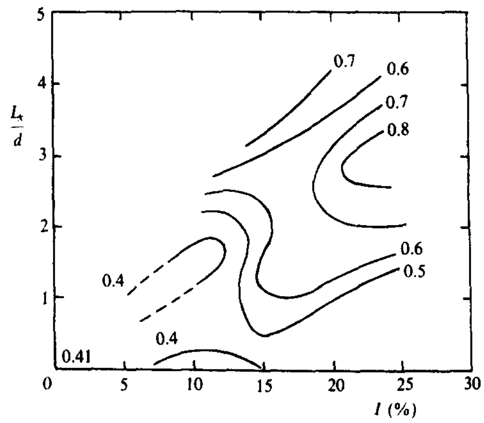

Figure 3 Drag coefficient contours: Re = 15,000 (approx.)

$C_D = 0.5$ is preserved for increasing turbulence intensities but only at increased scale values. Small scale, high intensity turbulence produces a reduced drag, much as would be expected from published material over many years. Large scale turbulence produces higher C_D at all intensities. Shown on the same figure is the location of the instability mentioned in Ref. 4, where drag coefficient was found to vary markedly while all other parameters were apparently held constant. The cause of this phenomenon is unknown but it was repeatable.

As Reynolds number decreases to 40×10^3 (Figure 2), the $C_D = 0.3$ region shrivels and the contours are generally lowered. Regions of very high drag, comparable with flat plate values, now appear at high intensities and large scale. At even lower Reynolds numbers (Figure 3), the $C_D = 0.3$ region is no longer discernible but a $C_D = 0.4$ finger is becoming evident, extending upwards from the origin. The highest drag values are now distinctly at high intensity, large scale combinations.

In the remaining two figures, 4 and 5, the diagonal low-drag area becomes more established as Reynolds number decreases to 6×10^3, the high-drag area being confined to higher intensities. Earlier in this paper, it was suggested that low-drag conditions could result from Van der Hegge Zijnen's resonance concept. In addition, if it could be assumed that a Strouhal number St_1 existed, based on L_x and u' for length and velocity scales, then Equation 1 predicted a low-drag area ought to occur around the line $L_x/d = C_1 \cdot I$. Figures 4 and 5 tend to confirm this reasoning; moreover, the slope C_1 would seem to be about 16, taking I as a true figure and not a percentage value. If St_2, based on downstream conditions, is taken to be about 0.2, then the value of St_1 ought to be about 3.2 (from Equation 1).

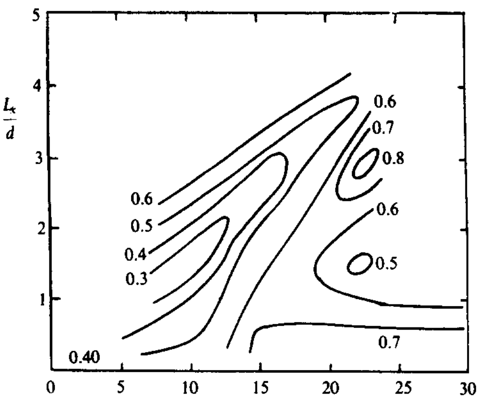

Figure 4 Drag coefficient contours: Re = 10,000 (approx.)

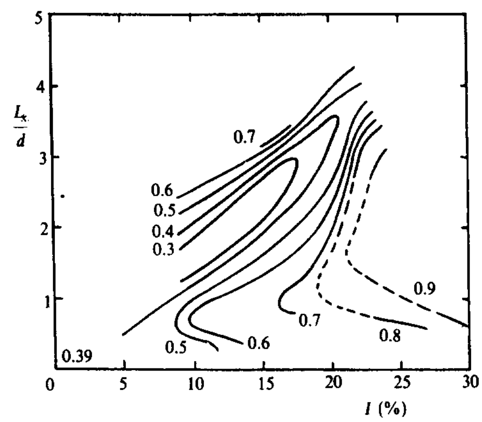

Figure 5 Drag coefficient contours: Re = 6,000 (approx.)

Whether this value is universal remains to be seen from other experimenters' findings.

Justification is needed for using results from grid-generated turbulence to make general statements concerning the effects of turbulence on sphere drag. No flow is truly isotropic in its turbulence characteristics, but Townsend[16] makes the point that the grid-generated variety is close enough to this ideal for most experimental results to be analyzed on the assumption of isotropy, provided the sensing position is not too close to the grids.

Pipe flow turbulence is shown by Laufer[17] to be nearly isotropic over the central section of the flow but not so near the walls, where the streamwise intensity climbs to about double the radial one, with the circumferential value between the two. Since the results of Torobin and Gauvin[1] and Clamen and Gauvin[2] were all obtained in pipe flows and since their data points were shown to line up with those of the present author[4] where their Reynolds numbers overlapped, it is reasonable to suggest that the current results are applicable to flows in pipes where spherical (or near-spherical) solids are being conveyed, turbulence intensities being taken from Laufer,[17] and macroscale levels from Neve.[18] A more accurate C_D value could then be used in pressure drop and settling velocity calculations. The use of a standard curve C_D value of 0.5 cannot be justifiable when the effective value might be as low as 0.3 or as high as 1.3.

Conclusions

Previous published results by many authors have shown that turbulent flows upstream and downstream of immersed spheres have coherent vortex structures, in each case with a characteristic Strouhal number; the former depends on the agency of turbulence production, the latter on sphere diameter and mean fluid speed. Van der Hegge Zijnen suggested that profound changes in heat transfer from a cylinder resulted from the upstream and downstream frequencies being comparable.

The current results tend to confirm that low drag coefficient conditions are repeatable when upstream turbulence characteristics bear some correct relationship to sphere diameter and Reynolds number, as given in Figures 1–5. In particular, if a Strouhal number St_1 for upstream conditions is based on L_x and u' then these results suggest a value of about 3.2 causes marked drag reduction.

At Reynolds numbers above about 20×10^3, when turbulence effects would be expected increasingly to be dominant over viscous ones, these low-drag conditions are restricted to small scale, high intensity flows and the reasoning which led to Equation 1 no longer seems applicable. Wake conditions then depend more closely on the turbulent state of the sphere's boundary layer than on conditions in the external flow.

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